ඌව පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය 2026

අවසාන වාර පරීක්ෂණය

13 ශ්‍රේණිය

සංයුක්ත ගණිතය 1

ලකුණු ලබා දීමේ පටිපාටිය

(01) $n=1$ විට

$L.H.S. = 1! = 1$

$R.H.S. = 2^{1-1} = 2^{0} = 1$ —— 5

$L.H.S = R.H.S$

$\therefore$ $n=1$ විට ප්‍රතිඵලය සත්‍ය වේ.

$n=p$; $p \in \mathbb{Z}^+$ සඳහා ප්‍රතිඵලය සත්‍ය යැයි උපකල්පනය කරමු.

$p! \geq 2^{p-1}$ —— 5

$n=p+1$ විට

$$(p+1)! = (p+1) \cdot p!$$

$$\geq (p+1)\, 2^{p-1} \quad \text{—— 5}$$

$$\geq 2 \cdot 2^{p-1} \quad \because \; p+1 \geq 2$$

$$\therefore (p+1)! \geq 2^{(p+1)-1}$$

$\therefore$ $n=p+1$ විට ප්‍රතිඵලය සත්‍ය වේ.

$n=1$ ට සත්‍ය විය. $n=p$ ට සත්‍ය යැයි උපකල්පනය 5 කළ විට $n=p+1$ ට ද සත්‍ය විය. $\therefore$ ගණිත අභ්‍යුහන මූලධර්මය අනුව සියලු $n \in \mathbb{Z}^+$ සඳහා $n! \geq 2^{n-1}$ වේ.

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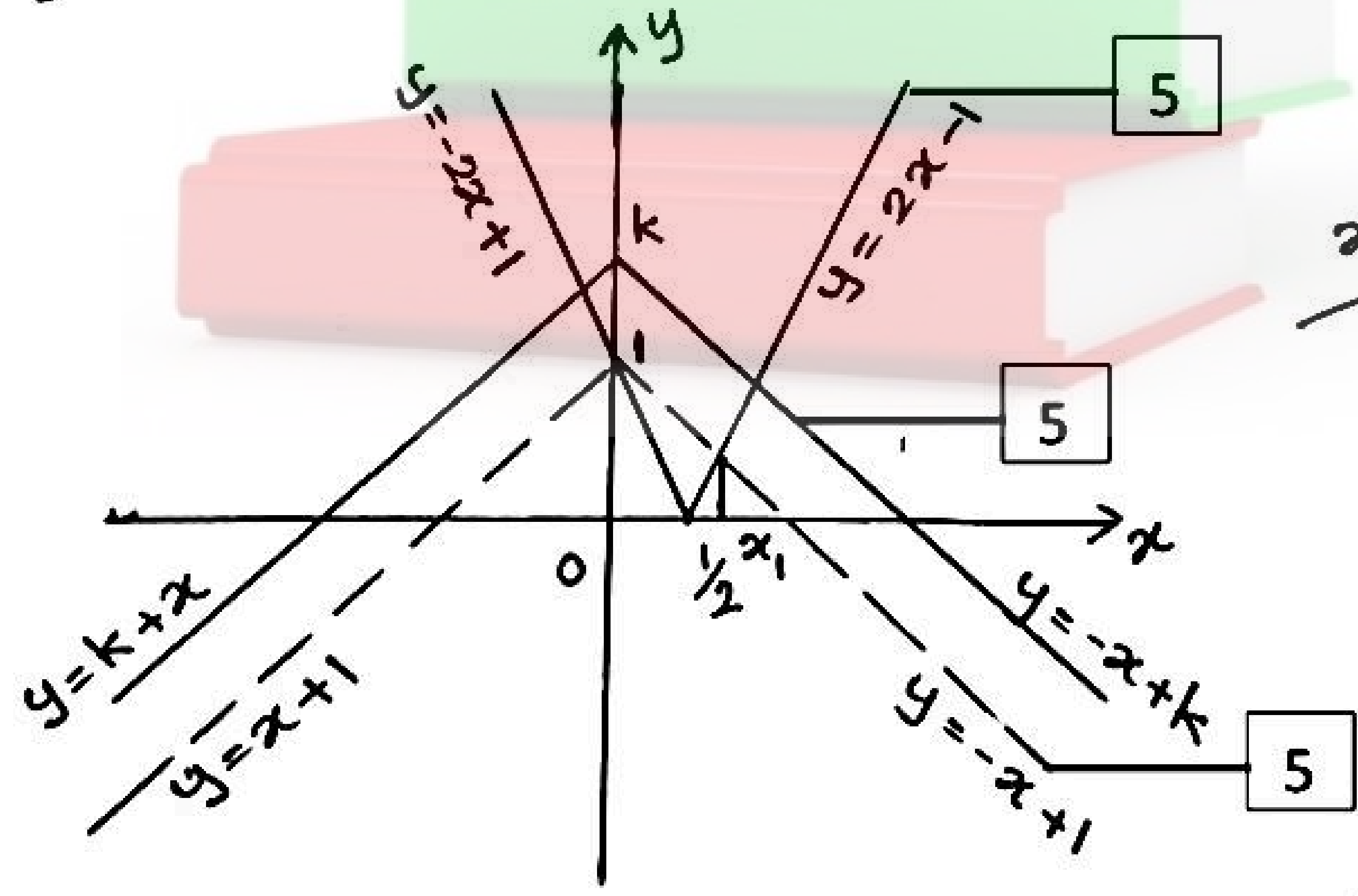
(02)



x_1

$y = 2x - 1$

$y = -x + 1$

$2x - 1 = -x + 1$

$3x = 2$

$x = \frac{2}{3}$ —— 5

$\therefore k = 1$

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(03) $2 \leqslant |\overline{z-1-i}| \leqslant 3$

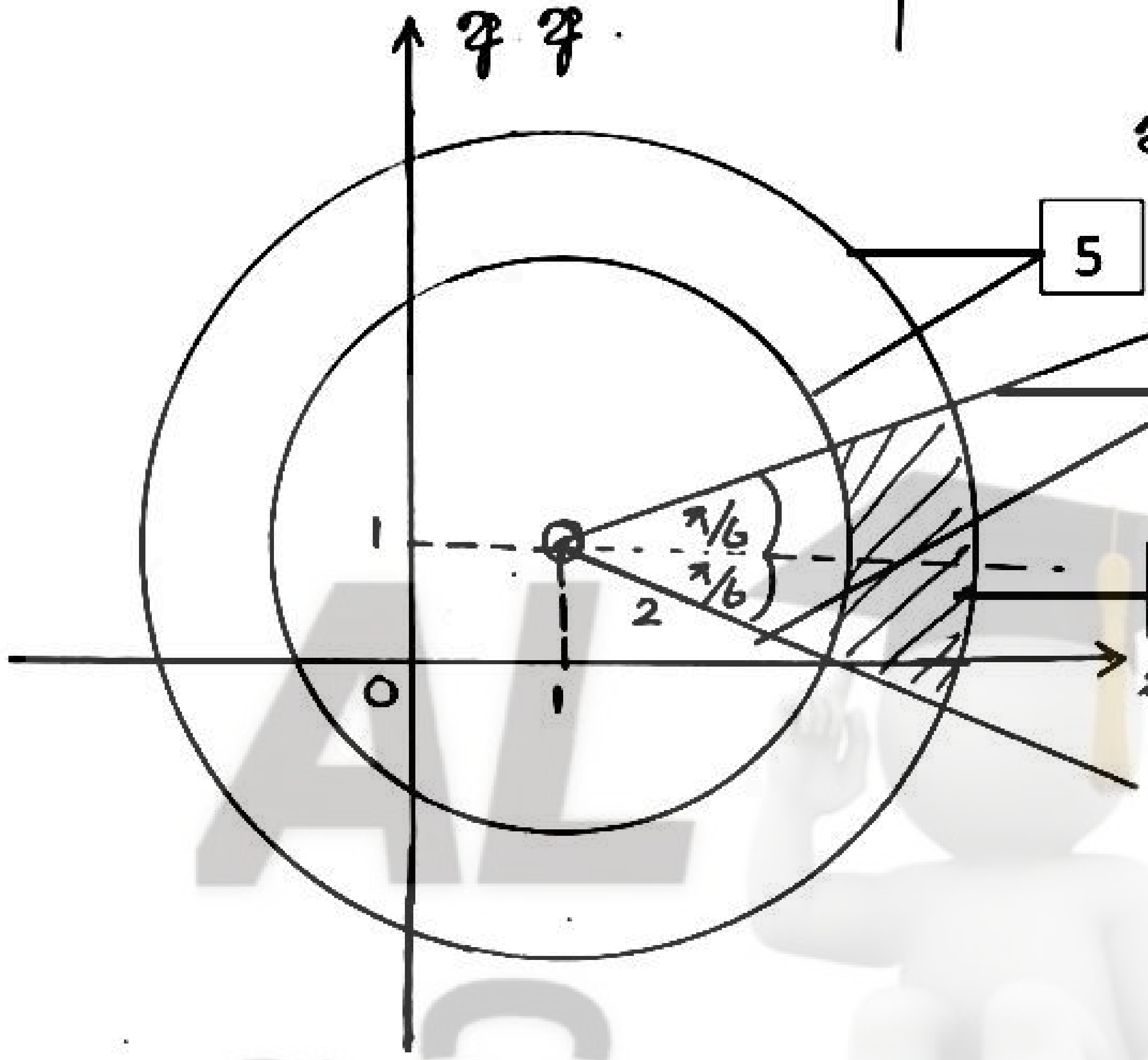
$2 \leqslant |\bar{z} - \overline{(1+i)}| \leqslant 3$

$2 \leqslant |\bar{z} - (1-i)| \leqslant 3$

$2 \leqslant |z - (1+i)| \leqslant 3$ — $\boxed{5}$

$|\text{Arg}(z-1-i)| \leqslant \pi/6$

$-\pi/6 \leqslant \text{Arg}(z-1-i) \leqslant \frac{\pi}{6}$ — $\boxed{5}$



$\text{ව.ඵ} = \frac{1}{2} \cdot 3^2 \cdot \frac{\pi}{3} - \frac{1}{2} 2^2 \cdot \frac{\pi}{3}$

$= \frac{\pi}{6}(9-4)$ — $\boxed{5}$

$= \frac{5\pi}{6}$ වර්ග ඒකක.

$\boxed{25}$

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(04) $(1+ax)^k = \sum_{r=0}^{k} {}^kC_r (ax)^r$ — $\boxed{5}$; ${}^kC_r = \frac{k!}{(k-r)!\,r!}$

; $0 \leq r \leq k$

x: ${}^kC_1 a = 8 \Rightarrow ka = 8$ — $\boxed{5}$

x^2: ${}^kC_2 a^2 = 30 \Rightarrow k(k-1)\frac{a^2}{2} = 30$ — $\boxed{5}$

$\frac{8}{a}\left(\frac{8}{a} - 1\right) a^2 = 60$

$64 - 8a = 60$

$a = 1/2$

$\therefore k = 16$ — $\boxed{5}$

x^3 හි සංගුණකය

$= {}^kC_3 a^3$

$= \frac{k(k-1)(k-2)a^3}{6}$

$= \frac{1}{6} \times 16 \times 15 \times 14 \times \frac{1}{8}$

$= 70$ — $\boxed{5}$

$\boxed{25}$

(5) $$\lim_{x\to 2} \frac{(x-2)^2}{\left(1+\cos\frac{\pi x}{2}\right)} \cdot \frac{\sin\pi(x-2)}{\left(\sqrt{x-1}-1\right)}$$

$$= \lim_{x\to 2} \frac{(x-2)^2\left(1-\cos\frac{\pi x}{2}\right)\sin\pi(x-2)\left(\sqrt{x-1}+1\right)}{\sin^2\frac{\pi x}{2}\;\; (x-1-1)} \quad \boxed{5} \quad \boxed{5}$$

$$= 2\times 2 \lim_{x\to 2} \frac{(x-2)^2}{\sin^2\left(\pi-\frac{\pi x}{2}\right)} \cdot \frac{\sin\pi(x-2)}{\pi(x-2)} \times \pi \quad \boxed{5}$$

$$= 4\pi \lim_{x\to 2} \frac{\left(\frac{\pi}{4}\right)^2(2-x)^2}{\sin^2\left[\frac{\pi}{2}(2-x)\right]} \times \frac{4}{\pi^2} \; \frac{\sin\pi(x-2)}{\pi(x-2)} \quad \boxed{5} \quad \boxed{5}$$

$$1 \qquad\qquad 1$$

$$= \frac{16}{\pi}$$

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(6)

$$f(x) = (x^2+1)e^{x^3+3x}$$

f හි පරාසය $\mathbb{R}^+$ $\boxed{5}$

පරිමාව $= \int_1^2 \pi y^2\, dx$ $\boxed{5}$

$$= \pi\int_1^2 (x^2+1)e^{x^3+3x}\,dx$$

$$= \frac{\pi}{3}\int_1^2 (3x^2+3)e^{x^3+3x}\,dx \quad \boxed{5}$$

$$= \frac{\pi}{3}\left[e^{x^3+3x}\right]_1^2 \quad \boxed{5}$$

$$= \frac{\pi}{3}\left[e^{14}-e^4\right] \quad \boxed{5}$$

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(07) $0 < t < \pi \qquad x = 5 + 4\cos t \ , \ y = 2 - \sin t$

$P \equiv \left(5 + 2\sqrt{2} \ , \ 2 - \frac{1}{\sqrt{2}}\right)$

P හිදී $t = \pi/4$ $\boxed{5}$ $\boxed{25}$

$\frac{dx}{dt} = -4\sin t \qquad \frac{dy}{dt} = -\cos t$

$\frac{dy}{dx} = \frac{\cot t}{4}$

$\left(\frac{dy}{dx}\right)_{t = \pi/4} = \frac{1}{4}$

P හිදී ස්පර්ශකය: $y - \left(2 - \frac{1}{\sqrt{2}}\right) = \frac{1}{4}\left(x - 5 - 2\sqrt{2}\right)$ $\boxed{5}$

$4y - 8 + 2\sqrt{2} = x - 5 - 2\sqrt{2}$

$x - 4y + 3 - 4\sqrt{2} = 0$

$5 + 4\cos t - 4(2 - \sin t) + 3 - 4\sqrt{2} = 0$

$4\cos t + 4\sin t = 4\sqrt{2}$

$\cos t + \sin t = \sqrt{2}$ $\boxed{5}$

$\cos(t - \pi/4) = 1 \implies t = 2n\pi + \pi/4$ $\boxed{5}$

$(0, \pi)$ තුළ $\pi/4$ හැර වෙනත් විසඳුම් නොමැත.

$\therefore$ නැවත වක්‍රය හමු නොවේ. $\boxed{5}$ $\boxed{25}$

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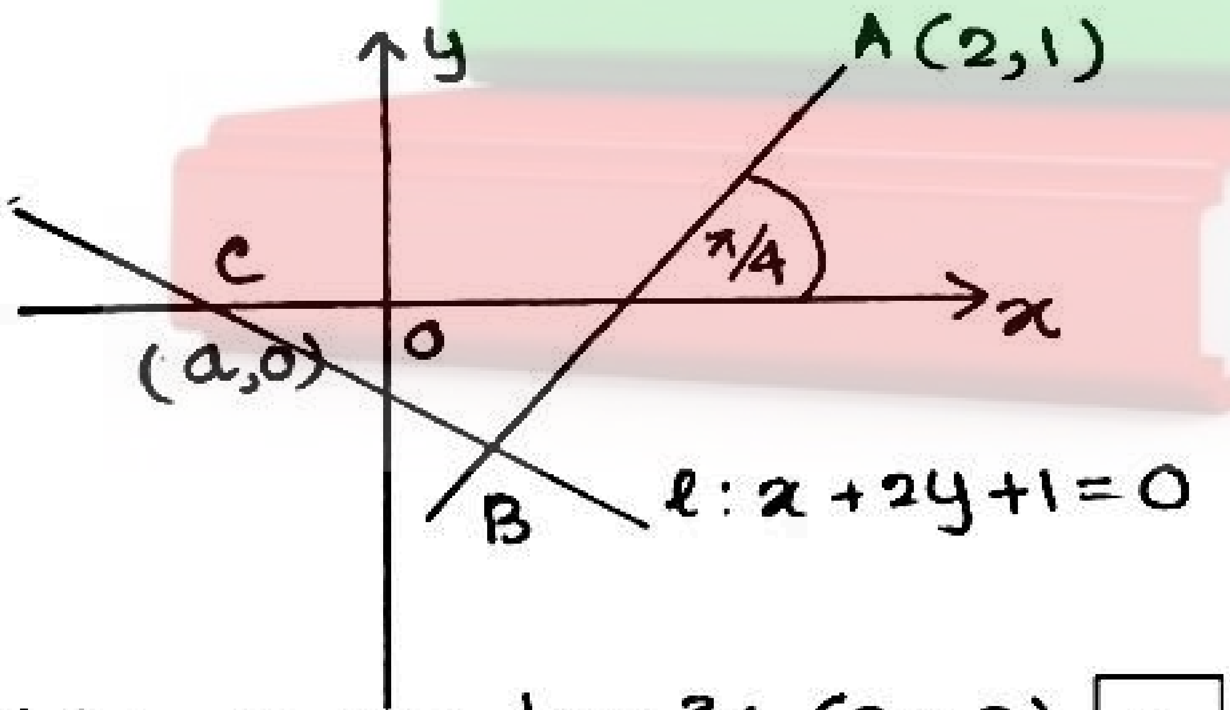
(08)



$B \equiv \left(\frac{1}{3}, -\frac{2}{3}\right)$ $\boxed{5}$

$C \equiv (a, 0)$, $x + 2y + 1 = 0$ මත නිසා

$a + 1 = 0 \implies a = -1$ $\boxed{5}$

AB: $y - 1 = \tan\frac{\pi}{4}(x - 2)$ $\boxed{5}$

$y - 1 = x - 2$

$x - y - 1 = 0$

$\left.\begin{array}{l} x - y - 1 = 0 \\ x + 2y + 1 = 0 \end{array}\right\} \quad 3y + 2 = 0$

$y = -2/3$

$x = 1/3$

$BC = \sqrt{\left(\frac{1}{3} + 1\right)^2 + \left(-\frac{2}{3} - 0\right)^2}$ $\boxed{5}$

$= \sqrt{\frac{16}{9} + \frac{4}{9}}$

$= \frac{2\sqrt{5}}{3}$ $\boxed{5}$ $\boxed{25}$

(09) $S=0$ හා $l=0$ හරහා යන ඕනෑම වෘත්තයක

සමීකරණය : $S + \lambda l = 0$

$x^2+y^2-6x+2y-17+\lambda(x-y+2)=0$

$x^2+y^2-(6-\lambda)x+(2-\lambda)y-17+2\lambda=0$ $\boxed{5}$

කේන්ද්‍රය $\left[\frac{6-\lambda}{2}, -\frac{(2-\lambda)}{2}\right]$ $\boxed{5}$, $l=0$ මත නිසා

$\frac{6-\lambda}{2} + \frac{2-\lambda}{2} + 2 = 0$

$2\lambda = 12$

$\lambda = 6$ $\boxed{5}$

$\therefore$ $S_1 : x^2+y^2-4y-5=0$ $\boxed{5}$

$S_1=0$ හි කේන්ද්‍රය $C_1=(0,2)$ $r_1=\sqrt{4+5}=3$

$S_2=0$ හි කේන්ද්‍රය $C_2=(3,-2)$ $r_2=2$

$C_1C_2=\sqrt{9+16}=5$ $\boxed{5}$

$C_1C_2=r_1+r_2$ නිසා $S_1=0$ හා $S_2=0$

වෘත්ත බාහිරව ස්පර්ශ වේ.

$\boxed{25}$

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(10) $\cos\theta-\sqrt{3}\sin\theta = 2\left\{\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right\}$ $\boxed{5}$

$= 2\left\{\cos\frac{\pi}{3}\cos\theta-\sin\frac{\pi}{3}\sin\theta\right\}$

$= 2\left\{\cos(\theta+\pi/3)\right\}$ $\boxed{5}$

$R=2$, $\alpha=\pi/3$ $\boxed{5}$

$\cos(\theta+\pi/3)=1$ විට $\cos\theta-\sqrt{3}\sin\theta$ උපරිම වේ.

$\cos(\theta+\pi/3)=1$ $\boxed{5}$

$\therefore$ $\cos(\theta+\pi/3)=\cos 0$

$\theta+\pi/3=2n\pi$; $n\in\mathbb{Z}$

$\theta = 2n\pi-\pi/3$ $\boxed{5}$

$\boxed{25}$

(11) a) $f(x) = x^2 - kx + k$

$$= \left(x - \frac{k}{2}\right)^2 + k - \frac{k^2}{4} \quad \boxed{5}$$

$$= \left(x - \frac{k}{2}\right)^2 + \frac{4k - k^2}{4}$$

$\left(x - \frac{k}{2}\right)^2 \geq 0$; සමානතාව $x = \frac{k}{2}$ විට $\boxed{5}$

$$\left(x - \frac{k}{2}\right)^2 + \frac{4k - k^2}{4} \geq \frac{k(4-k)}{4}$$

$$f(x) \geq \frac{k(4-k)}{4}$$

$$\therefore f(x)_{min} = \frac{k(4-k)}{4} \quad \boxed{5}$$

$f(x)_{min} < 0$ $\boxed{5}$

$k(4-k) < 0$

$\Rightarrow k < 0$ or $k > 4$ $\boxed{5}$

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$\alpha > 0$ හා $\beta > 0$ නම්

$\alpha + \beta > 0$ හා $\alpha\beta > 0$ $\boxed{5}$

$k > 0$ හා $k > 0$

$\therefore k > 0$ $\boxed{5}$

10

$$(\sqrt{\alpha} + \sqrt{\beta})^2 = \alpha + \beta + 2\sqrt{\alpha\beta} \quad \boxed{5}$$

$$= k + 2\sqrt{k}$$

$$\sqrt{\alpha} + \sqrt{\beta} = \sqrt{k + 2\sqrt{k}} \quad \boxed{5}$$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} \quad \boxed{5}$$

$$= \frac{\sqrt{k + 2\sqrt{k}}}{\sqrt{k}} \quad \boxed{5}$$

$$\frac{1}{\sqrt{\alpha}} \cdot \frac{1}{\sqrt{\beta}} = \frac{1}{\sqrt{\alpha\beta}}$$

$$= \frac{1}{\sqrt{k}} \quad \boxed{5}$$

$\therefore$ සමීකරණය :

$$\left(x - \frac{1}{\sqrt{\alpha}}\right)\left(x - \frac{1}{\sqrt{\beta}}\right) = 0 \quad \boxed{5}$$

$$x^2 - \left(\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}}\right)x + \frac{1}{\sqrt{\alpha}\sqrt{\beta}} = 0 \quad \boxed{5}$$

$$x^2 - \frac{\sqrt{k + 2\sqrt{k}}}{\sqrt{k}}\,x + \frac{1}{\sqrt{k}} = 0 \quad \boxed{5}$$

$$\boxed{40}$$

$$\sqrt{k}\,x^2 - \sqrt{k + 2\sqrt{k}}\;x + 1 = 0$$

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b) පොදු මූලය α නම්

$$a\alpha^2 + b\alpha + 1 = 0 \text{ --- } (1)$$

$$b\alpha^2 + a\alpha + 1 = 0 \text{ --- } (2) \quad \boxed{5}$$

$(1) - (2)$ $\quad (a-b)\alpha^2 + (b-a)\alpha = 0$

$$(a-b)\alpha(\alpha - 1) = 0$$

$\therefore \alpha = 0$ හෝ $\alpha = 1$

$$\alpha \neq 0 \Rightarrow \alpha = 1 \quad \boxed{5}$$

$$\therefore a + b + 1 = 0 \quad \boxed{5}$$

$$2x^3 + (a+b)x^2 + (a+b+3)x = (a+b)^2$$

$$2x^3 - x^2 + 2x = (-1)^2$$

$$2x^3 - x^2 + 2x - 1 = 0 \quad \boxed{5}$$

$$x^2(2x-1) + (2x-1) = 0$$

$$(x^2+1)(2x-1) = 0$$

$$\therefore x = \tfrac{1}{2} \quad \boxed{5}$$

$$\boxed{25}$$

c) $p(x) = (x-k)^3 Q(x)$ $\boxed{5}$

$p'(x) = (x-k)^3 Q'(x) + Q(x) \cdot 3(x-k)^2$

$p'(x) = (x-k)^2 \left[(x-k)Q'(x) + 3Q(x)\right]$ $\boxed{5}$ $\boxed{10}$

$\therefore (x-k)^2$ යනු $p'(x)$ හි සාධකයකි.

$f(x) = 2x^4 - 5x^3 + 3x^2 + ax + b$

$f(1) = 0$ $\boxed{5}$

$2(1)^4 - 5(1)^3 + 3(1)^2 + a(1) + b = 0$

$a + b = 0$ $\boxed{5}$

$f'(x) = 8x^3 - 15x^2 + 6x + a$ $\boxed{5}$

$f'(1) = 0$ $\boxed{5}$ $\Rightarrow 8 - 15 + 6 + a = 0$

$a = 1$

$\therefore \quad b = -1$ $\boxed{5}$

$2x^4 - 5x^3 + 3x^2 + ax + b = (x-1)^3(2x+q)$

$2x^4 - 5x^3 + 3x^2 + x - 1 = (x-1)^3(2x+q)$ $\boxed{5}$

$x^0 \Rightarrow -1 = -q \Rightarrow q = 1$ $\boxed{5}$

$\therefore$ අනෙක් සාධකය $(2x+1)$

$\boxed{40}$

(12) a) (10) ← 5   (5) ← 3

i) (10) කාසි දෙක හා (5) කාසි 2 ලබාදිය හැකි ආකාර ගණන $= {}^{6}C_{2} \cdot 2!$ [5] [5]

ඉතිරි (10) කාසි 3 හා (5) කාසිය ලබාදිය හැකි ආකාර ගණන $= {}^{4}C_{1}$

$\therefore$ මුළු ආකාර ගණන $= {}^{4}C_{1} \times {}^{6}C_{2} \times 2!$ [5]

$= 120$ [5]

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<u>හෝ</u>

${}^{6}C_{1} \times {}^{5}C_{1} \times {}^{4}C_{3} \times {}^{1}C_{1} = 120$ [20]

ii) ${}^{6}C \times {}^{5}C_{2} \times {}^{3}C_{3} = 6 \times 10 \times 1 = 60$ [5X4=20]

iii)

සංයුක්තිය (කාසි 3 ලැබෙන සිසුවා)		ආකාර ගණන		
(10)	(5)			
3	0	${}^{6}C_{1} \times {}^{5}C_{2} \times {}^{3}C_{3}$	$6 \times 10 \times 1$	5
–	3	${}^{6}C_{1} \times {}^{5}C_{5}$	6×1	5
2	1	${}^{6}C_{1} \times {}^{5}C_{3} \times {}^{2}C_{2}$	$6 \times 10 \times 1$	5
1	2	${}^{6}C_{1} \times {}^{5}C_{4} \times {}^{1}C_{1}$	$6 \times 5 \times 1$	5

මුළු ආකාර ගණන $= 60 + 6 + 60 + 30$

$= \underline{\underline{156}}$ [5]

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(12) b) $U_r = \dfrac{r^2+11r+5}{r^2(r+1)^2}$

$$\frac{1}{r} + \frac{5}{r^2} - \frac{1}{r+1} - \frac{5}{(r+1)^2} = \frac{r(r+1)^2+5(r+1)^2-1(r+1)r^2-5r^2}{r^2(r+1)^2}$$

$$= \frac{r^3+2r^2+r+5r^2+10r+5-r^3-r^2-5r^2}{r^2(r+1)^2} \quad \boxed{5}$$

$$= \frac{r^2+11r+5}{r^2(r+1)^2} \quad \boxed{5}$$

$$\therefore \frac{r+5}{r^2} - \frac{(r+1)+5}{(r+1)^2} = U_r \quad \boxed{5}$$

$$V_r - V_{r+1} = U_r \ ; \ V_r = \frac{r+5}{r^2} \quad \boxed{5} \qquad \boxed{20}$$

$$U_r = V_r - V_{r+1}$$

$$U_1 = V_1 - \cancel{V_2} \quad \boxed{5}$$

$$U_2 = \cancel{V_2} - V_3$$

$$\vdots \qquad \vdots$$

$$U_{n-1} = V_{n-1} - \cancel{V_n} \quad \boxed{5}$$

$$U_n = \cancel{V_n} - V_{n+1}$$

$$\sum_{r=1}^{n} U_r = V_1 - V_{n+1} \quad \boxed{5}$$

$$= 6 - \frac{(n+1)+5}{(n+1)^2} = 6 - \frac{n+6}{(n+1)^2} \quad \boxed{5}$$

$$\sum_{r=1}^{\infty} U_r = \lim_{n\to\infty} \sum_{r=1}^{n} U_r$$

$$= \lim_{n\to\infty} 6 - \frac{n+6}{(n+1)^2} \quad \boxed{5}$$

$$= 6 - 0$$

$= 6$ (පරිමිතයි) $\boxed{5}$

$\therefore \sum_{r=1}^{\infty} U_r$ ශ්‍රේණිය අභිසාරී වේ $\boxed{5}$

$$\sum_{r=1}^{\infty} U_r = 6 \quad \boxed{5} \qquad \boxed{40}$$

(12) Cont...

$$\sum_{r=1}^{n-1} U_{n-r} = U_{n-1} + U_{n-2} + \cdots + U_2 + U_1$$

$$\sum_{r=3}^{n} U_r = U_3 + U_4 + \cdots + U_{n-1} + U_n.$$

$$\sum_{r=1}^{n-1} U_{n-r} + \sum_{r=3}^{n} U_r = 2\left(U_3 + U_4 + \cdots + U_{n-1}\right) + U_1 + U_2 + U_n \quad \boxed{5}$$

$$= 2\sum_{r=1}^{n} U_r - \left(U_1 + U_2 + U_n\right) \quad \boxed{5}$$

$$\lim_{n\to\infty}\left[\sum_{r=1}^{n-1} U_{n-r} + \sum_{r=3}^{n} U_r\right] = \lim_{n\to\infty} 2\sum_{r=1}^{n} U_r - \lim_{n\to\infty}\left(U_1 + U_2 + U_n\right) \quad \boxed{5}$$

$$= 2\sum_{r=1}^{\infty} U_r - U_1 - U_2 - \lim_{n\to\infty} U_n$$

$$= 2\times 6 - \frac{1+11+5}{1(1+1)^2} - \frac{4+22+5}{4\cdot 3^2} \quad \boxed{5}$$

$$= 12 - \frac{17}{4} - \frac{31}{36}$$

$$= \frac{432 - 153 - 31}{36}$$

$$= \frac{248}{36} \quad \boxed{5}$$

$$= \frac{62}{9}$$

$\boxed{25}$

(13) a) $A = \begin{pmatrix} 1 & -a \\ a & 1 \end{pmatrix}$

$\det A = 1 + a^2 \neq 0$ $\boxed{5}$

$\therefore A^{-1}$ පවතී. $\boxed{5}$ **10**

$AA^T = \begin{pmatrix} 1 & -a \\ a & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ -a & 1 \end{pmatrix}$ $\boxed{5}$

$= \begin{pmatrix} 1+a^2 & a-a \\ a-a & a^2+1 \end{pmatrix}$

$= (a^2+1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\boxed{5}$

$= (a^2+1)\, I$

$A\left(\dfrac{A^T}{1+a^2}\right) = I$ $\boxed{5}$

$\therefore A^{-1} = \dfrac{A^T}{1+a^2} = \dfrac{1}{1+a^2}\begin{pmatrix} 1 & a \\ -a & 1 \end{pmatrix}$ $\boxed{5}$

$A^{-1} = \begin{pmatrix} \dfrac{1}{1+a^2} & \dfrac{a}{1+a^2} \\ \dfrac{-a}{1+a^2} & \dfrac{1}{1+a^2} \end{pmatrix}$ $\boxed{5}$ **25**

$a = 2$ විට

$A + 2I = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\therefore B = \begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$ $\boxed{5}$ $\quad B^{-1} = \dfrac{\begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}}{13}$ $\boxed{5}$

$2A - 3R = \left(\dfrac{1}{13}B\right)^{-1}$

$2A - 3R = 13B^{-1}$ $\boxed{5}$

$\therefore 3R = 2A - 13B^{-1} \Rightarrow \begin{pmatrix} 2 & -4 \\ 4 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$

$\therefore R = \begin{pmatrix} -1/3 & -2 \\ 2 & -1/3 \end{pmatrix}$ $\boxed{5}$ **20**

(13) b) $Z_1 = -\sqrt{3} + i \quad , \quad W = 1 - i$

$$Z = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \boxed{5} \qquad W = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) \boxed{5}$$

$$Z = 2\left(\cos\left(\pi - \pi/6\right) + i \sin\left(\pi - \pi/6\right)\right)$$

$$= 2\left[\cos\frac{5\pi}{6} + i \sin\frac{5\pi}{6}\right] \boxed{5}$$

$$W = \sqrt{2}\left[\cos\left(-\pi/4\right) + i \sin\left(-\pi/4\right)\right] \boxed{5}$$

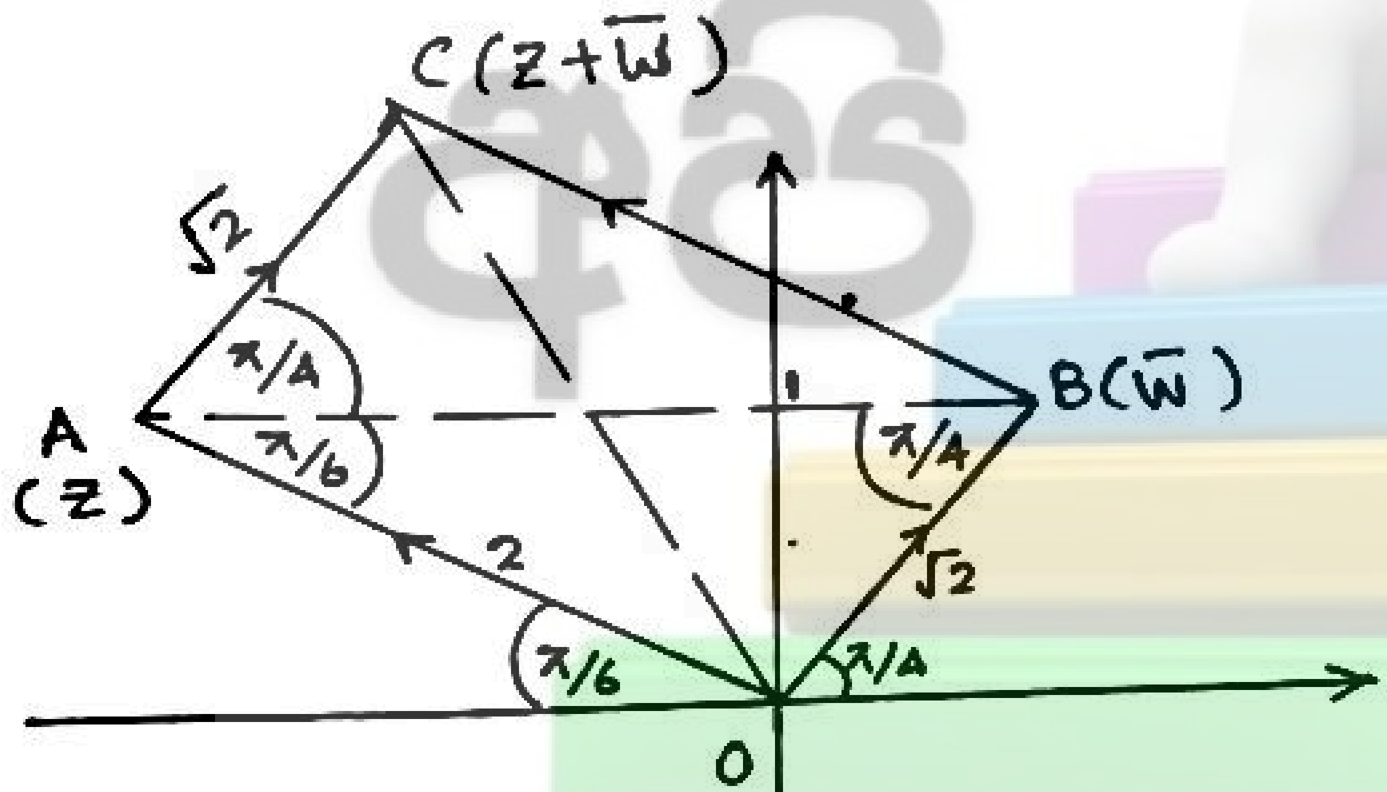
$$|Z| = 2 \;\boxed{5}\;, \quad |W| = \sqrt{2} \;, \quad \mathrm{Arg}(Z) = \frac{5\pi}{6} \;, \quad \mathrm{Arg}(W) = \frac{-\pi}{4} \;\boxed{5}$$

$\boxed{30}$

$$Z + \bar{W} = -\sqrt{3} + i + 1 + i$$

$$= (1 - \sqrt{3}) + 2i \quad \boxed{5}$$

$$|Z + \bar{W}| = \sqrt{(1-\sqrt{3})^2 + 2^2} = \sqrt{8 + 2\sqrt{3}} \quad \boxed{5}$$



$\boxed{10}$

$$A\hat{O}B = \pi - \pi/6 - \pi/4$$

$$= \pi - \frac{5\pi}{12} = \frac{7\pi}{12} \quad \boxed{5}$$

$$\cos A\hat{O}B = \cos\frac{7\pi}{12}$$

$$\cos(A\hat{O}B) = \frac{2^2 + (\sqrt{2})^2 - (1+\sqrt{3})^2}{2.2.\sqrt{2}} \quad \boxed{5}$$

$$\therefore \cos\frac{7\pi}{12} = \frac{4 + 2 - 1 - 2\sqrt{3} - 3}{4\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}} \quad \boxed{5}$$

$$|Z + \bar{W}| = OC$$

OAC Δ හි $\quad O\hat{A}C = 5\pi/12 \quad \boxed{5}$

$$OC^2 = 2^2 + (\sqrt{2})^2 - 2.2\sqrt{2}\cos\frac{5\pi}{12}$$

$$= 4 + 2 + 4\sqrt{2}\,\frac{(1-\sqrt{3})}{2\sqrt{2}}$$

$$= 4 + 2 + 2 + 2\sqrt{3}$$

$$= 8 + 2\sqrt{3} \quad \boxed{5}$$

$$\therefore |Z + \bar{W}| = \sqrt{8 + 2\sqrt{3}}$$

$\boxed{45}$

13) c) $(\cot\theta + i)^n + (\cot\theta - i)^n$

$$= \left(\frac{\cos\theta}{\sin\theta} + i\right)^n + \left(\frac{\cos\theta}{\sin\theta} - i\right)^n$$

$$= \frac{1}{\sin^n\theta}\left[(\cos\theta + i\sin\theta)^n + (\cos\theta - i\sin\theta)^n\right]$$

$$= \operatorname{Cosec}^n\theta\left[\cos n\theta + i\sin n\theta + (\cos(-\theta) + i\sin(-\theta))^n\right] \boxed{5}$$

$$= \operatorname{Cosec}^n\theta\left[\cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)\right]$$

$$= \operatorname{Cosec}^n\theta\left[\cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta\right] \boxed{5}$$

$$= 2\operatorname{Cosec}^n\theta \cos n\theta$$

$$\left(\cot\frac{\pi}{20} + i\right)^{10} + \left(\cos\frac{\pi}{20} - i\right)^{10}$$

$$= 2\operatorname{Cosec}^{10}\frac{\pi}{20}\cos\frac{10\pi}{20} \boxed{5}$$

$$= 2\operatorname{Cosec}^{10}\frac{\pi}{20}\cos\frac{\pi}{2}$$

$$= 2 \cdot \operatorname{Cosec}^{10}\pi/20 \cdot 0$$

$$= \underline{\underline{0}} \boxed{5}$$

$\boxed{20}$

---

14) a) $f(x) = \dfrac{2x^2 + px}{(x+3)^2}$, $f'(1) = \dfrac{5}{32}$ $\boxed{5}$

$$f'(x) = \frac{(x+3)^2(4x+p) - (2x^2+px)\,2(x+3)}{(x+3)^4 \boxed{5}} \boxed{5}$$

$$= \frac{(x+3)(4x+p) - 2(2x^2+px)}{(x+3)^3}$$

$$= \frac{\cancel{4x^2} + px + 12x + 3p - \cancel{4x^2} - 2px}{(x+3)^3}$$

$$= \frac{(12-p)x + 3p}{(x+3)^3} \boxed{5}$$

$\boxed{20}$

$$f'(1) = \frac{5}{32} \Rightarrow \frac{(12-p).1 + 3p}{64} = \frac{5}{32}$$ 5

$$12 - p + 3p = 10$$

$$2p = -2$$

$$p = -1$$ 5

10

$$\therefore f(x) = \frac{2x^2 - x}{(x+3)^2}$$ 5 $$f'(x) = \frac{13x - 3}{(x+3)^3}$$

10

$$f'(x) = 0 \Rightarrow x = 3/13$$ 5

	$x < -3$	$-3 < x < 3/13$	$x > 3/13$
$f'(x)$	$(+)$	$(-)$	$(+)$
x සමග $f(x)$	වැඩිවේ	අඩුවේ	වැඩිවේ.

5X3=15

10

$x = 3/13$ විට $$f(x) = \frac{2 \cdot (3/13)^2 - 3/13}{(3/13 + 3)^2}$$

$$= \frac{\frac{3}{\cancel{13^2}}(6 - 13) \times \cancel{13^2}}{42 \times 42}$$

$$= \frac{3 \times (-7)}{42 \times 42} = \frac{-1}{84}$$

$\therefore (3/13, -1/84)$ ස්ථානීය අවම ලක්ෂ්‍යයකි. 5

30

$$f''(x) = 0 \Leftrightarrow 24 - 13x = 0$$

$$x = 24/13$$ 5

	$3/13 < x < 24/13$	$x > 24/13$
$f''(x)$	$+$	$-$
	උඩු අවතල	යටි අවතල

5

5

$x = 24/13$ දී නත්වර්තන ලක්ෂ්‍යයක් පවතී.

5

20

(14) Cont... $f(x) = \dfrac{2x^2 - x}{(x+3)^2} = \dfrac{x(2x-1)}{(x+3)^2}$

$\lim_{x \to \pm\infty} f(x) = 2$

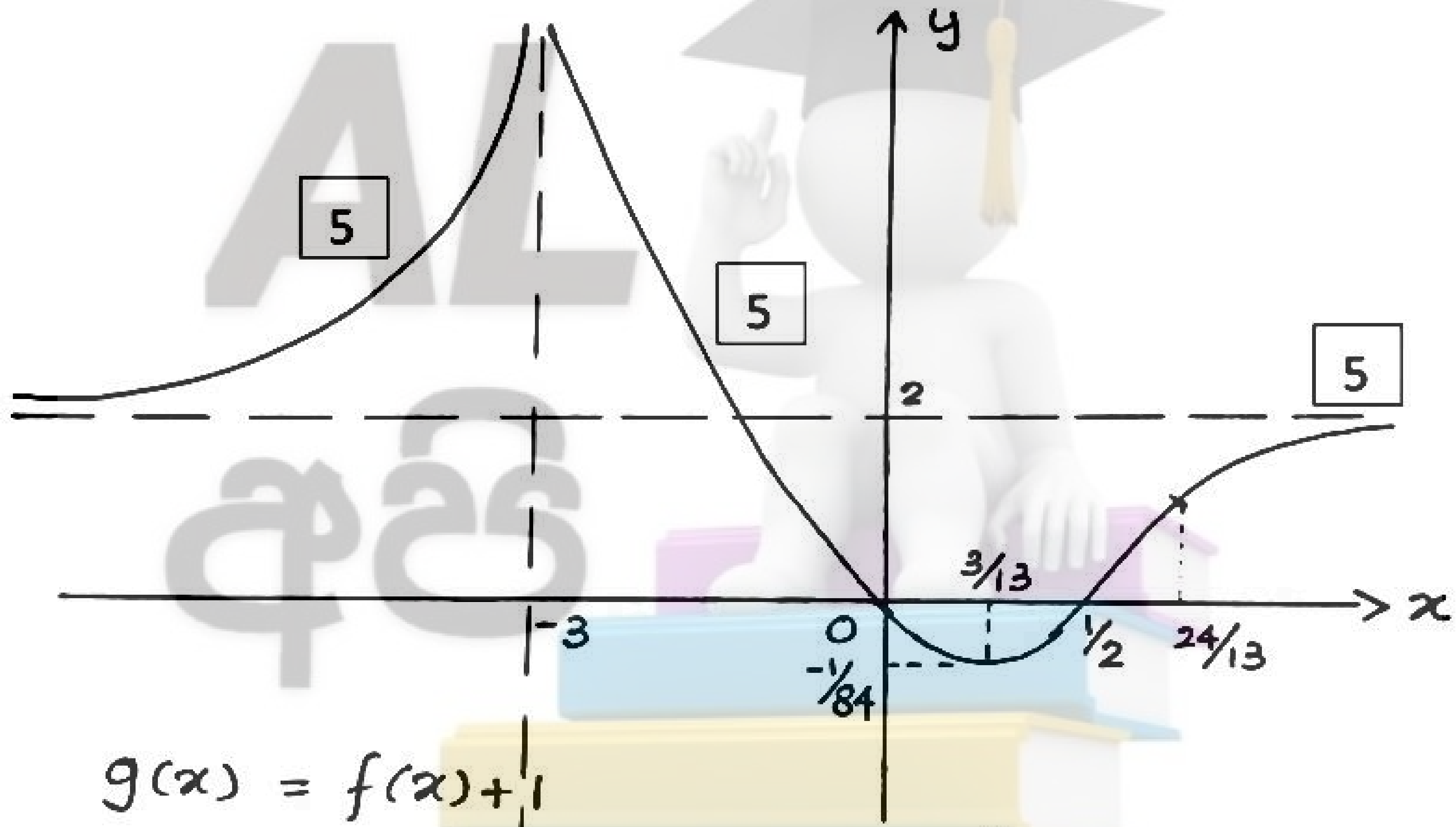
$\therefore$ $y = 2$ තිරස් ස්පර්ශෝන්මුඛයකි. $\boxed{5}$

$x \to -3^-$ විට $f(x) \to \infty$

$x \to -3^+$ විට $f(x) \to \infty$

$\therefore$ $x = -3$ සිරස් ස්පර්ශෝන්මුඛයකි. $\boxed{5}$

$f(0) = 0$ $f(x) = 0 \Rightarrow x = 0, x = \frac{1}{2}$



$\boxed{25}$

$g(x) = f(x) + 1$

$y = g(x)$ හි අවම ලක්ෂ්‍යය $\left(\frac{3}{13}, \frac{83}{84}\right)$ $\boxed{5}$ $\boxed{05}$

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b) $A = 2\pi(2r)2x + 2\pi r\left(2x - \frac{x}{2}\right) + \pi(2r)^2 + \pi(2r)^2$ $\boxed{5}$

$= 8\pi r x + 3\pi r x + 4\pi r^2 + 4\pi r^2$

$= 11\pi r x + 8\pi r^2$ $\boxed{5}$

$x = \dfrac{A - 8\pi r^2}{11\pi r}$

පරිමාව $V = \pi r^2\left(2x - \frac{x}{2}\right) = \frac{3}{2}\pi r^2 x$ $\boxed{5}$

$V = \frac{3}{2}\pi r^2\left(\dfrac{A - 8\pi r^2}{11\pi r}\right)$

$$V = \frac{3}{22} r \left(A - 8\pi r^2\right)$$

$$\frac{dV}{dr} = \frac{3}{22}\left[A - 24\pi r^2\right] \quad \boxed{5}$$

$$\frac{dV}{dr} = 0 \Longleftrightarrow \quad r^2 = \frac{A}{24\pi}$$

$\boxed{5}$

$$r > 0 \Rightarrow \quad r = \sqrt{\frac{A}{24\pi}} \quad \boxed{5}$$

$0 < r < \sqrt{A/24\pi}$ විට $\frac{dV}{dr} > 0$

$r > \sqrt{A/24\pi}$ විට $\frac{dV}{dr} < 0$ $\boxed{5}$

$\therefore$ $r = \sqrt{A/24\pi}$ විට V උපරිම වේ. $\boxed{5}$

$A = 6\pi$ විට $r = \sqrt{\frac{6\pi}{24\pi}} = \frac{1}{2}$ $\boxed{5}$

$$x_{(r=1/2)} = \frac{6\pi - 8\pi \cdot 1/4}{11\pi \cdot 1/2} \quad \boxed{5}$$

$$= \frac{4 \times 2}{11}$$

$$= \frac{8}{11}$$

$\boxed{50}$

(15) a) $\dfrac{x^2+x-1}{x^2(x-1)} = \dfrac{A(x-1)+Bx(x-1)+Cx^2}{x^2(x-1)}$

$x^2+x-1 = A(x-1)+Bx(x-1)+Cx^2$

$x^2 \Rightarrow \quad B+C = 1$

$x \Rightarrow \quad A-B = 1$

$x^0 \Rightarrow \quad -A = -1 \quad \Rightarrow A=1,\ B=0,\ C=1$ $\boxed{5X3=15}$

$\int \dfrac{\sin\theta\cos\theta - \cos^3\theta}{(1-\cos^2\theta)(\sin\theta-1)}\,d\theta$ හි $x = \sin\theta$ ආදේශ.

$dx = \cos\theta\, d\theta$ $\boxed{5}$

$\int \dfrac{x-(1-x^2)}{x^2(x-1)}\,dx = \int \dfrac{x^2+x-1}{x^2(x-1)}\,dx$ $\boxed{5}$

$= \int \dfrac{(x-1)+x^2}{x^2(x-1)}\,dx$

$= \int \dfrac{1}{x^2} + \dfrac{1}{x-1}\,dx$ $\boxed{5}$

$= \dfrac{-1}{x} + \ln|x-1| + c$ $\boxed{5}$

$= \dfrac{-1}{\sin\theta} + \ln|\sin\theta-1| + C.$

$\boxed{5}$ $\boxed{5}$ $\boxed{5}$ $\boxed{50}$

---

b) $x = \dfrac{1}{2}(1+\tan\theta) \Rightarrow \tan\theta = 2x-1$

$2\,dx = \sec^2\theta\, d\theta$ $\boxed{5}$

$x=0$ විට $\theta = -\pi/4$

$x=1$ විට $\theta = \pi/4$ $\boxed{5}$

(15) Cont...

$$\int_0^1 \frac{2x-1}{\sqrt{(2x-1)^2+1}}\,dx$$

$$= \int_{-\pi/4}^{\pi/4} \frac{\left[2\cdot\frac{1}{2}(\tan\theta+1)-1\right]\frac{1}{2}\sec^2\theta\,d\theta}{\sqrt{\left[2\times\frac{1}{2}(\tan\theta+1)-1\right]^2+1}} \quad \boxed{5}$$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} \frac{\tan\theta\sec^2\theta}{\sqrt{\tan^2\theta+1}}\,d\theta \quad \boxed{5}$$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} \tan\theta\sec\theta\,d\theta \quad \boxed{5}$$

$$= \frac{1}{2}\left[\sec\theta\right]_{-\pi/4}^{\pi/4} \quad \boxed{5}$$

$$= \frac{1}{2}\left(\sqrt{2}-\sqrt{2}\right) \quad \boxed{5}$$

$$= 0 \quad \boxed{5}$$

$\boxed{40}$

---

c) $I = \int x\sin(\ln|x|)\,dx \qquad J = \int x\cos(\ln|x|)\,dx$

$I = \frac{1}{2}\int \sin(\ln|x|)\frac{d}{dx}x^2\,dx \quad \boxed{5} \qquad J = \frac{1}{2}\int \cos(\ln|x|)\frac{d}{dx}x^2\,dx \quad \boxed{5}$

$2I = \left[x^2\sin(\ln|x|)\right] - \int x\cos(\ln|x|)\,dx \quad \boxed{5}$

$2I + J \boxed{5} = x^2\sin(\ln|x|) + c$ —— (1) $\boxed{5}$

$2J \boxed{5} = x^2\cos(\ln|x|) + \int x\sin(\ln|x|)\,dx \quad \boxed{5}$

$2J - I = x^2\cos(\ln|x|) + c'$ —— (2) $\boxed{5}$

(1) හා (2) න් $\quad 5I = 2x^2\sin(\ln|x|) - x^2\cos(\ln|x|) \quad \boxed{5}$

$I = \frac{2}{5}x^2\sin(\ln|x|) - \frac{x^2}{5}\cos(\ln|x|) + C_1 \quad \boxed{5}$

$5J = x^2\sin(\ln|x|) + 2x^2\cos(\ln|x|) \quad \boxed{5}$

$J = \frac{x^2}{5}\sin(\ln|x|) + \frac{2}{5}x^2\cos(\ln|x|) + C_2 \quad \boxed{5}$

$\boxed{60}$

(16) $2x - y + 3 = 0 \quad , \quad x - 2y - 1 = 0$

$$\frac{|2\bar{x} - \bar{y} + 3|}{\sqrt{5}} = \frac{|\bar{x} - 2\bar{y} - 1|}{\sqrt{5}} \quad \boxed{5}$$

$$2x - y + 3 = \pm (x - 2y - 1)$$

$(+) \Rightarrow \quad x + y + 4 = 0 \quad \boxed{5}$

$(-) \Rightarrow \quad 3x - 3y + 2 = 0 \quad \boxed{5}$

$x + y + 4 = 0 \Rightarrow m_1 = -1$

$2x - y + 3 = 0 \Rightarrow m_2 = 2$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-1-2}{1-2}\right| \quad \boxed{5}$$

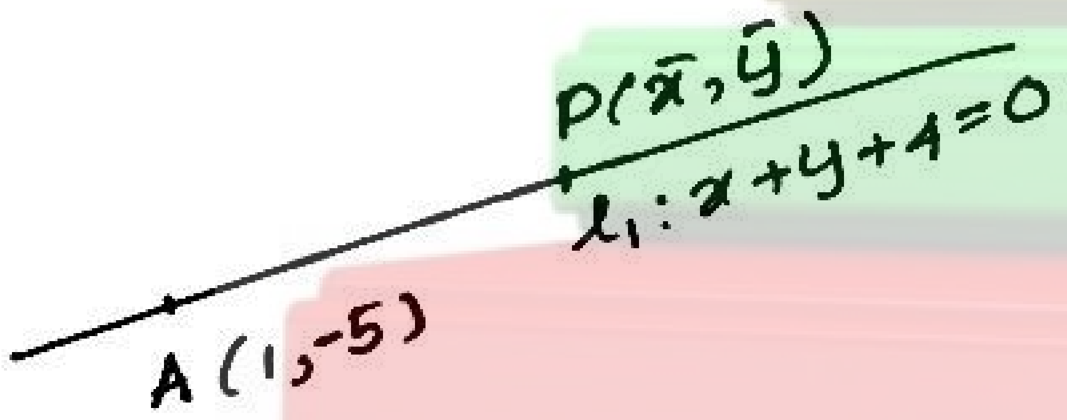
$$= 3 > 1 \quad \boxed{5}$$

$\therefore x + y + 4 = 0$ මහා කෝණ සමච්ඡේදකය වේ. $\boxed{5}$

$A = (1, -5)$ ආදේ. $x + y + 4 \Rightarrow 1 - 5 + 4 = 0 \quad \boxed{5}$

$\therefore A \equiv (1, -5)$ මහා කෝණ සමච්ඡේදකය මත වේ.

$\boxed{5}$ $\boxed{40}$



$$\frac{\bar{y} + 5}{\bar{x} - 1} = -1 \quad \boxed{5}$$

$\bar{x} - 1 = \dfrac{\bar{y} + 5}{-1} = t$ ගනිමු. $\boxed{5}$

$\bar{x} = 1 + t$

$\bar{y} = -5 - t$

$\therefore l_1 = 0$ මත ඕනෑම ලක්ෂ්‍යයක් $(1+t, -5-t)$ ලෙස ලිවිය හැකිය. $\boxed{5}$

$B = (1+t, -5-t)$ ලෙස ලිවිය හැකිය. $\boxed{15}$

$AB = \sqrt{2}$

$AB^2 = (1+t-1)^2 + (-5-t+5)^2$ $\boxed{5}$

$2 = 2t^2$

$t = \pm 1$ $\boxed{5}$

$(+) \Rightarrow (2, -6)$ $\boxed{5}$ $\quad (-) \Rightarrow (0, -4)$ $\boxed{5}$

y අක්ෂය මත පිහිටයි $\quad B \equiv (0, -4)$

$A \equiv (1, -5) \quad , \quad B \equiv (0, -4)$ $\boxed{5}$ $\qquad \boxed{25}$

$\therefore \quad (x-1)^2 + (y+5)^2 = 2$ $\boxed{5}$

$x^2 - 2x + 1 + y^2 + 10y + 25 = 2$

$S: \; x^2 + y^2 - 2x + 10y + 24 = 0$ $\boxed{5}$ $\qquad \boxed{10}$

$S = 0 \Rightarrow g_1 = -1, \; f = +5, \; c = 24$

$S_1' = x^2 + y^2 - r^2 = 0 \Rightarrow g_1 = f_1 = 0, \quad c_2 = -r^2$

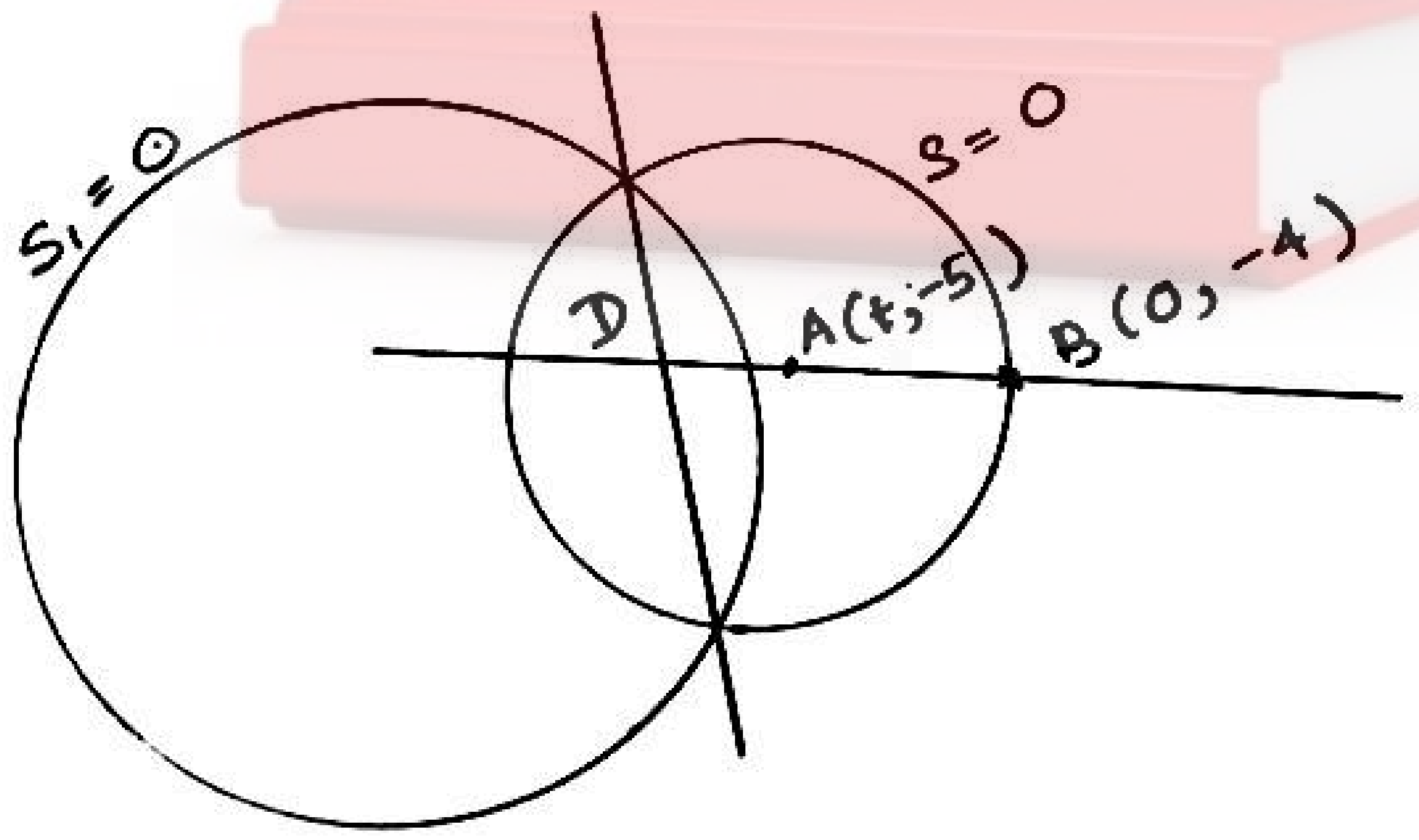
ප්‍රලම්බව ඡේදනය වේ නම්

$2gg_1 + 2ff_1 = c_1 + c_1$ $\boxed{5}$

$0 + 0 = 24 - r^2$

$r^2 = 24$ $\boxed{5}$

$\therefore \quad S_1: \; x^2 + y^2 - 24 = 0$ $\boxed{5}$ $\qquad \boxed{15}$



$x + y + 4 = 0$

$x - 5y - 24 = 0$

$6y = -28$

$y = -14/3$

$x = 2/3$

$\therefore \; D \equiv \left(2/3, \; -14/3\right)$

$\boxed{5}$

පොදු ජ්‍යාය:

$S_1 - S = 0$ $\boxed{5}$

$2x - 10y - 24 = 24$

$x - 5y - 24 = 0$ $\boxed{5}$ $\qquad \boxed{15}$

(16) Cont...

$P \equiv (x_0, y_0)$ ගනිමු.

PD හි මධ්‍ය ලක්ෂ්‍යය $(\bar{x}, \bar{y})$ නම්

$$\bar{x} = \frac{x_0 + 2/3}{2} = \frac{3x_0 + 2}{6} \Rightarrow x_0 = \frac{6\bar{x} - 2}{3}$$

$$\bar{y} = \frac{y_0 - 14/3}{2} \boxed{5} = \frac{3y_0 - 14}{6} \Rightarrow y_0 = \frac{6\bar{y} + 14}{3} \boxed{5}$$

$P(x_0, y_0)$, $S_1 = 0$ මත නිසා

$$x_0^2 + y_0^2 = 24 \quad \boxed{5}$$

$$\left(\frac{6\bar{x} - 2}{3}\right)^2 + \left(\frac{6\bar{y} + 14}{3}\right)^2 = 24 \quad \boxed{5}$$

$$36\bar{x}^2 - 24\bar{x} + 4 + 36\bar{y}^2 + 2.14.6\bar{y} + 14.14 = 24 \times 9$$

$$9\bar{x}^2 - 6\bar{x} + 1 + 9\bar{y}^2 + 42\bar{y} + 49 - 54 = 0$$

$$9\bar{x}^2 + 9\bar{y}^2 - 6\bar{x} + 42\bar{y} - 4 = 0$$

$$\bar{x}^2 + \bar{y}^2 - \frac{2}{3}\bar{x} + \frac{14}{3}\bar{y} - \frac{4}{9} = 0 \quad \boxed{5}$$

$(\bar{x}, \bar{y}) \rightarrow (x, y) \Rightarrow x^2 + y^2 - 2/3\, x + 14/3\, y - 4/9 = 0$

$\therefore$ මධ්‍ය ලක්ෂ්‍යයේ පථය කේන්ද්‍රය $(1/3, -7/4)$ $\boxed{5}$

වන වෘත්තයක් වේ.

$\boxed{30}$

(17) a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\boxed{5}$

$A = B = \theta$ යෙදූ විට

$\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$ $\boxed{5}$

$= 2\sin\theta\cos\theta$ $\boxed{5}$

$\boxed{15}$

$\cot\theta - 3\tan\theta = \sin 2\theta$

$\dfrac{\cos\theta}{\sin\theta} - 3\dfrac{\sin\theta}{\cos\theta} = 2\sin\theta\cos\theta$ $\boxed{5}$

$\cos^2\theta - 3\sin^2\theta = 2\sin^2\theta\cos^2\theta$

$1 - \sin^2\theta - 3\sin^2\theta = 2\sin^2\theta(1-\sin^2\theta)$ $\boxed{5}$

$1 - 4\sin^2\theta - 2\sin^2\theta + 2\sin^4\theta = 0$

$2\sin^4\theta - 6\sin^2\theta + 1 = 0$ $\boxed{5}$

$a = 2,\ b = -6,\ c = 1$ $\boxed{5}$

$\boxed{20}$

$x = \sin^2\theta$ ගනිමු.

$2x^2 - 6x + 1 = 0$ $\boxed{5}$

$x = \dfrac{6 \pm \sqrt{36-8}}{4}$

$= \dfrac{3 \pm \sqrt{7}}{2}$ $\boxed{5}$

$\sin^2\theta = \dfrac{3-\sqrt{7}}{2} \quad \because \quad 0 < x < 1$ $\boxed{5}$

$\sin\theta = \pm\sqrt{\dfrac{3-\sqrt{7}}{2}}$ $\boxed{5}$

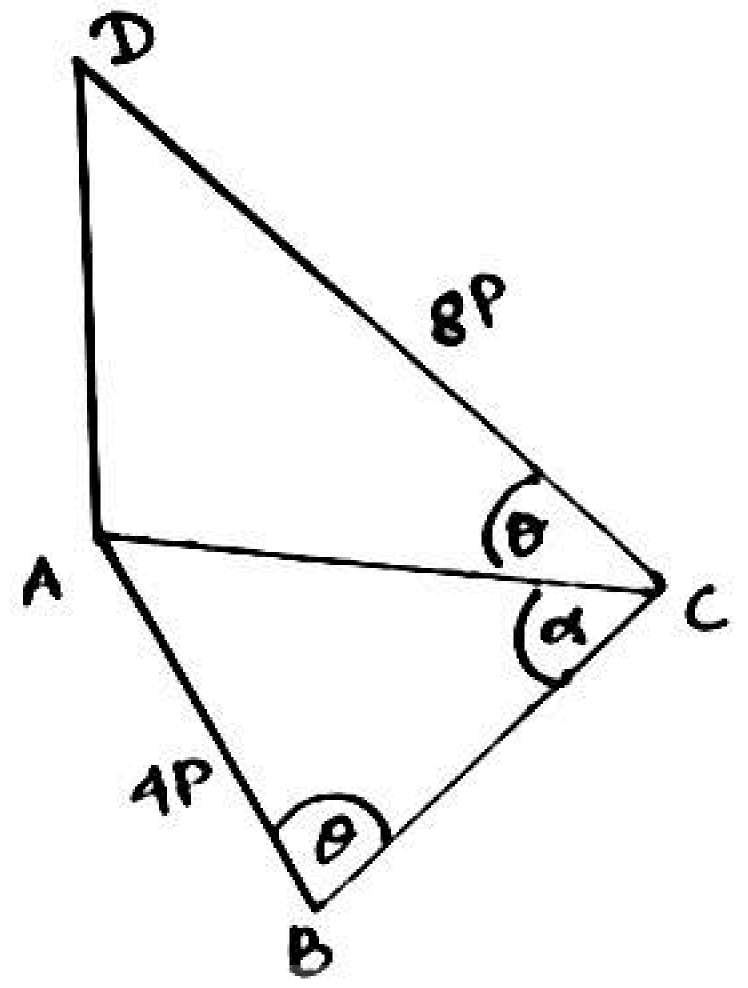
$0 < \theta < \pi/2$ නිසා

$\sin\theta = \sqrt{\dfrac{3-\sqrt{7}}{2}}$ $\boxed{5}$

$\boxed{25}$

$\theta = \sin^{-1}\sqrt{\dfrac{3-\sqrt{7}}{2}}$

b)



ABC Δ ට සයින නියමයෙන්

$$\frac{AB}{\sin\alpha} = \frac{AC}{\sin\theta} \quad \boxed{5}$$

$$\frac{4P}{\sin\alpha} = \frac{AC}{\sin\theta} \quad \boxed{5}$$

$$AC = \frac{4P\sin\theta}{\sin(\pi/2-\theta)}$$

$$= 4P\frac{\sin\theta}{\cos\theta} \quad \boxed{5}$$

$\boxed{15}$

$$AC = 4P\tan\theta$$

ACD Δ ට Cos නියමයෙන්

$$AD^2 = AC^2 + CD^2 - 2\,AC\cdot CD\cos\theta \quad \boxed{5}$$

$$= 16P^2\tan^2\theta + 64P^2 - 2(4P\tan\theta)8P\cos\theta \quad \boxed{5}$$

$$= 16P^2\tan^2\theta + 64P^2 - 64P^2\tan\theta\cos\theta$$

$$AD = 8P$$

$$64P^2 = 16P^2\tan^2\theta + 64P^2 - 64P^2\tan\theta\cos\theta \quad \boxed{5}$$

$$16P^2\tan\theta(\tan\theta - 4\cos\theta) = 0 \quad \boxed{5}$$

$\boxed{20}$

$$\tan\theta \neq 0 \Rightarrow \tan\theta - 4\cos\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} - 4\cos\theta = 0$$

$$\sin\theta - 4(1-\sin^2\theta) = 0$$

$$4\sin^2\theta + \sin\theta - 4 = 0 \quad \boxed{5}$$

$$\sin\theta = \frac{-1 \pm \sqrt{1+64}}{8}$$

$$= \frac{-1 \pm \sqrt{65}}{8} \quad \boxed{5}$$

$$0 < \theta < \pi/2 \Rightarrow \sin\theta > 0 \quad \boxed{5}$$

$$\therefore \sin\theta = \frac{\sqrt{65}-1}{8} \Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{65}-1}{8}\right) \quad \boxed{5}$$

$\boxed{20}$

e) $\sin^{-1}x + \sin^{-1}y = \pi/3$

$\sin^{-1}x = \alpha \quad , \quad \sin^{-1}y = \beta \quad \Rightarrow \sin\alpha = x, \quad \sin\beta = y$ $\boxed{5}$

$\alpha + \beta = \pi/3 \quad \Rightarrow \quad \alpha = \pi/3 - \beta$

$\sin\alpha = \sin(\pi/3 - \beta)$ $\boxed{5}$

$\sin\alpha = \sin\pi/3 \cos\beta - \cos\pi/3 \sin\beta$ $\boxed{5}$

$x = \frac{\sqrt{3}}{2}\sqrt{1-y^2} - \frac{1}{2}y$ $\boxed{5}$

$2x + y = \sqrt{3}\sqrt{1-y^2}$ $\boxed{5}$

$(2x+y)^2 = 3(1-y^2)$ $\boxed{5}$

$4x^2 + 4xy + y^2 = 3 - 3y^2$

$4x^2 + 4y^2 = 3 - 4xy$ $\boxed{5}$ $\boxed{35}$